Comparing two Interior-Point Approaches for Semi-Infinite Programs

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Abstract. In several attempts to generalize interior-point approaches to semi-infinite programs it has been observed that barrier methods based on the integral over some barrier function exhibit poor local convergence properties.

This observation contrasts the intuition that, typically, semi-infinite programs possess a piecewise smooth boundary just like finite dimensional optimization problems for which interior-point methods have proved to be very efficient.

In this paper we show that even for simple examples the integral over a barrier function cannot provide a self-concordant barrier function for a semi-infinite program. We discuss an alternative implicit formulation of a barrier function that shares many properties with barrier functions for finite dimensional optimization problems.

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1. Introduction

Many important applications result in optimization problems with finitely many design variables and infinitely many constraints, see e.g. [1, 2, 3, 8, 9], and the references therein. In several earlier attempts [4, 10, 11] to generalize interior-point approaches to semi-infinite programs it has been observed that barrier methods based on the integral over some barrier function exhibit poor local convergence properties. It appears that the efforts to adapt interior-point methods to semi-infinite problems have stopped since then, and that up to now, the design of an interior-point method that proves to be of practical importance for solving such problems has not yet been presented.

In this paper we provide some insight in the difference of barrier methods for finitely many constraints and infinitely many constraints.

To illustrate the main idea of our approach we consider the following simple problem

$$\text{minimize} \quad c^T x \quad \text{s.t.} \quad a(t)^T x \le b(t) \quad \text{for} \quad t \in \Omega. \tag{1}$$

Here $c \in \mathbb{R}^n$, $a(t) \in \mathbb{R}^n$ for $t \in \Omega$, and $b(t) \in \mathbb{R}$ for $t \in \Omega$ are the data, and x is the unknown. For the present paper we assume a very simple form where Ω is the interval

$$\Omega = [0, 1].$$

We denote the feasible set of (1) by

$$\mathcal{S} := \{x \in \mathbb{R}^n \mid \ a(t)^T x \le b(t) \ \text{ for } \ t \in \Omega\}. \tag{2}$$

We note that in the context of interior-point methods, there is no conceptual difficulty in forming the intersection of several domains of the form (2). Hence, if Ω is the union of several intervals, all statements of this paper are still valid. For the sake of simplicity we make the following assumption.

Assumption 1 *The interior int($\mathcal{S}$) is not empty.*

Let

$$\phi : \mathbb{R}^+ \to \mathbb{R}$$

be a barrier function for the positive real axis. We require that ϕ is convex, monotone, and satisfies $\lim_{t\to 0,\ t>0} \ \phi(t) = \infty$. In the case of finitely many constraints ($\Omega = \{t_1, \ldots, t_m\}$ for some fixed number m), the choice $\phi(r) = -\log r$ results in a very suitable barrier function of the form $\sum_{i=1}^m \phi(b(t_i) - a(t_i)^T x)$.

For $\Omega = [0, 1]$ and $x \in \text{int}(\mathcal{S})$ we define a "barrier" function[1] φ by

$$\varphi(x) := \int_0^1 \phi\left(b(t) - a(t)^T x\right) \ dt. \tag{3}$$

For sufficiently smooth ϕ and continuous a, b, we obtain

$$\nabla\varphi(x) = -\int_0^1 a(t) \cdot \phi'\left(b(t) - a(t)^T x\right) \ dt,$$

$$\nabla^2\varphi(x) = \int_0^1 a(t) \ a(t)^T \cdot \phi''\left(b(t) - a(t)^T x\right) \ dt,$$

and

$$\nabla^3\varphi(x)[h, h, h] = \int_0^1 \left(a(t)^T h\right)^3 \cdot \phi'''\left(b(t) - a(t)^T x\right) \ dt.$$

The self-concordance conditions

$$\nabla^3\varphi(x)[h, h, h] \le 2\nabla^2\varphi(x)[h, h]^{3/2}$$

and

$$\nabla\varphi(x)[h] \le \left(\vartheta \nabla^2\varphi(x)[h, h]\right)^{1/2}$$

for some $\vartheta \ge 1$ are essential in the analysis of interior-point methods for convex programs. If both conditions are satisfied for any $x \in \text{int}(\mathcal{S})$ and any $h \in \mathbb{R}^n$, then the theory by Nesterov

[1]We write "barrier" in quotes since, as we schall se below, the barrier property $\lim_{t\to 0,\ t>0} \ \varphi(t) = \infty$ may get lost.

and Nemirovskii [7] guarantees "rapid" convergence of a barrier method using the function φ. Conversely, if φ cannot be scaled (multiplied by a positive constant) such that these conditions hold for some $\vartheta \geq 1$, there is little reason to hope that a barrier method based on the function φ will be numerically efficient.

The example in Section 2 shows, no matter how we choose the barrier function ϕ defining φ, the self-concordance conditions can never be guaranteed, implying that a barrier approach based on (3) is very unlikely to be numerically efficient.

In contrast, if the set of "nearly active" points t is suitably approximated, we obtain a barrier function of very different nature. This is shown in the Section 3.

2. Self-concordance of the integral barrier function

The self-concordance conditions can be written as

$$\left(\int_0^1 \left(a(t)^T h\right)^3 \cdot \phi''' \left(b(t) - a(t)^T x\right)\, dt\right)^2 \leq 4 \left(\int_0^1 \left(a(t)^T h\right)^2 \cdot \phi'' \left(b(t) - a(t)^T x\right)\, dt\right)^3 \quad (4)$$

and

$$\left(\int_0^1 a(t)^T h \cdot \phi' \left(b(t) - a(t)^T x\right)\, dt\right)^2 \leq \vartheta \int_0^1 \left(a(t)^T h\right)^2 \cdot \phi'' \left(b(t) - a(t)^T x\right)\, dt \quad (5)$$

for some fixed $\vartheta \geq 1$. If we can identify a class of (smooth) functions a, b such that both inequalities hold for all points $x \in \operatorname{int}(\mathcal{S})$ and all $h \in I\!R^n$, then, the theory of self-concordant barrier functions can be applied.

If (4) and (5) are to hold for a nontrivial class of constraint functions a, b, we would expect them to hold at least for the following example.

A simple example in $I\!R^2$:
Let $\mathcal{S} := \{x \in I\!R^2 \mid a(t)^T x \leq b(t)\}$ with

$$a(t) = \begin{pmatrix} -(t-\frac{1}{2})^2 \\ -1 \end{pmatrix} \quad \text{and} \quad b(t) \equiv 0. \quad (6)$$

For this example we consider two cases:

1.) In the first case we choose

$$x = x_\epsilon = \begin{pmatrix} 0 \\ \epsilon \end{pmatrix}, \quad h = \begin{pmatrix} 0 \\ -1 \end{pmatrix},$$

for arbitrary $\epsilon > 0$. This results in $a(t)^T h \equiv 1$, and $b(t) - a(t)^T x \equiv \epsilon$. Note that $x_{\epsilon=0} \notin \operatorname{int}(\mathcal{S})$.

2.) In the second case we choose

$$\tilde{x} = \tilde{x}_\epsilon = \begin{pmatrix} 1 \\ \epsilon \end{pmatrix}, \quad h = \begin{pmatrix} 0 \\ -1 \end{pmatrix},$$

for arbitrary $\epsilon > 0$. This results in $a(t)^T h \equiv 1$, and $b(t) - a(t)^T \tilde{x}_\epsilon \equiv \epsilon + (t - \frac{1}{2})^2$. For $\tilde{t} = \frac{1}{2}$ we find $b(\tilde{t}) - a(\tilde{t})^T \tilde{x}_\epsilon = \epsilon$ implying that $\tilde{x}_{\epsilon=0} \notin \operatorname{int}(\mathcal{S})$.

In the first case the conditions (4) and (5) result in

$$\left(\int_0^1 \phi'''(\epsilon)\ dt\right)^2 \leq 4\left(\int_0^1 \phi''(\epsilon)\ dt\right)^3 \quad \text{and} \quad \left(\int_0^1 \phi'(\epsilon)\ dt\right)^2 \leq \vartheta \int_0^1 \phi''(\epsilon)\ dt$$

for some fixed $\vartheta \geq 1$, which reduces to

$$(\phi'''(\epsilon))^2 \leq 4(\phi''(\epsilon))^3 \quad \text{and} \quad (\phi'(\epsilon))^2 \leq \vartheta\phi''(\epsilon)$$

saying that ϕ is a ϑ-self-concordant barrier function. For ϑ-self-concordant barrier functions of the positive real axis it follows from [7, Equations (2.3.1) and (2.3.7)] that

$$-\frac{\vartheta}{t} \leq \phi'(t) \leq -\frac{1}{t} \quad \text{for} \quad t > 0. \tag{7}$$

Without loss of generality we may always assume that $\phi(1) = 0$, and thus, by integration, the inequalities (7) imply

$$\phi(1+t) \in \begin{cases} [-\log(1+t),\ -\vartheta\log(1+t)] & \text{for } t \in (-1,0), \\ [-\vartheta\log(1+t),\ -\log(1+t)] & \text{for } t \geq 0, \end{cases} \tag{8}$$

saying that $\phi(t)$ "closely resembles" a multiple of the function $-\log t$.

In the second case from above, it follows from (8) for $\epsilon < \frac{3}{4}$ (for such ϵ it is $\epsilon + (t-\frac{1}{2})^2 < 1$ for $t \in [0,1]$) the relation

$$\begin{aligned} \varphi(\tilde{x}_\epsilon) &= \int_0^1 \phi(\epsilon + (t-\frac{1}{2})^2)\ dt \\ &\leq \int_0^1 -\vartheta\log(\epsilon + (t-\frac{1}{2})^2)\ dt \\ &= -2\vartheta\int_{1/2}^1 \log(\epsilon + (t-\frac{1}{2})^2)\ dt \\ &\leq -2\vartheta\int_{1/2}^1 \log((t-\frac{1}{2})^2)\ dt \\ &= -4\vartheta\int_{1/2}^1 \log(t-\frac{1}{2})\ dt = 2\vartheta\log 2, \end{aligned} \tag{9}$$

i.e. $\varphi(\tilde{x}_\epsilon)$ remains bounded as $\tilde{x}_\epsilon$ approaches the boundary of $\mathrm{int}(\mathcal{S})$ for $\epsilon \to 0$. So, as mentioned before, φ looses the barrier property. In particular, in the notation of [7], φ is not strongly self-concordant, and hence not a ϑ-self-concordant barrier function for any $\vartheta \geq 1$. We summarize this observation in a theorem.

Theorem 1 *For any $\vartheta \geq 1$, there is no ϑ-self-concordant barrier function φ of the form* (3) *for the set $\mathcal{S}$ defined by* (6).

Discussion

Note that in the second case where $\tilde{x}_\epsilon = (1,\epsilon)^T$, the value t minimizing $d_{x_\epsilon}(t) = b(t) - a(t)^T\tilde{x}_\epsilon$ is $t = \frac{1}{2}$ and lies in the middle of Ω. There are even simpler examples where $d_{x_\epsilon}(t)$ is minimized at one of the boundary points ($t = 0$ or $t = 1$) of Ω. However, for such examples, one might argue that a density function μ which gives some extra weight to the end points of Ω could possibly result in a more suitable barrier function $\tilde{\varphi}(x) := \int_0^1 \phi(b(t) - a(t)^T x)\ d\mu$. We therefore chose an example in case 2.) where the boundary of Ω is inactive as $\epsilon \to 0$.

We also note that for $t \in [0, \frac{1}{2}]$ the vectors $a(\frac{1}{2}+t)$ and $a(\frac{1}{2}-t)$ coincide. This feature is not essential for the derivation of Theorem 1; the theorem also holds for simple examples like

$$a(t) = \begin{pmatrix} -t \\ -1 \end{pmatrix} \quad \text{and} \quad b(t) \equiv 0$$

with $x_\epsilon = (0,\epsilon)^T$, $h = (0,-1)^T$, and

$$a(t) = \begin{pmatrix} 1 \\ t \end{pmatrix} \quad \text{and} \quad b(t) = 1 + t^2$$

with $\tilde{x}_\epsilon = 4(\sqrt{5}-2)(1-\epsilon)(1,\frac{1}{2})^T$, $h = (1,0)^T$, where $\mathcal{S}$ has a smooth nonlinear boundary. The computations are slightly more involved.

More generally, condition (8) above can be deduced from examples where $b(t) + \epsilon$ is a linear combination of $a_i(t)$, while the boundedness of the integral log barrier function in (9) is true under very general circumstances.

We now consider a class of problems where we exclude the case where $b(t) + \epsilon$ is a linear combination of $a_i(t)$. First, we consider an example where $x \in \text{int}(\mathcal{S})$ and Δx is such that $x + \lambda\Delta x \in \text{int}(\mathcal{S})$ for $\lambda > -1$, and $x + \lambda\Delta x \notin \mathcal{S}$ for $\lambda < -1$. (When $\mathcal{S}$ is unbounded (and $\mathcal{S} \neq I\!R^n$), then for any $x \in \text{int}(\mathcal{S})$ we can find such Δx.) We define $x_\epsilon = x + (\epsilon - 1)\Delta x$ and assume that

$$b(t) - a(t)^T x_\epsilon = f(\epsilon) + \Theta(t - \bar{t})^2 \tag{10}$$

for some $\bar{t} \in [0,1]$. Here, f is a differentiable function with $f(0) = 0$ and $f'(0) > 0$, and the symbol Θ stands for the notion

$$x = \Theta(y) \qquad \Longleftrightarrow \qquad \exists \text{ constant } c \geq 1 \text{ with } \frac{1}{c}y \leq x \leq cy.$$

The situation (10) can be considered the typical case when approaching the boundary of $\mathcal{S}$. It occurs, for example, for the constraint $x_2 \geq x_1^2$ when represented in a semi-infinite form via $-2tx_1 + x_2 \geq -t^2$ and when t is restricted to $t \in [-1,1]$. In this case $x_\epsilon = (0,\epsilon)$ is of the above form.

Since $\varphi(x_\epsilon)$ as a function of ϵ is a self-concordant barrier function for $\{\epsilon \in I\!R \mid \epsilon \geq 0\}$ it follows as in (8) that

$$\varphi(x_\epsilon) \in [-\log\epsilon, -\vartheta\log\epsilon], \tag{11}$$

when $\epsilon \leq 1$. (For $\epsilon > 1$ it is $\varphi(x_\epsilon) \in [-\vartheta\log\epsilon, -\log\epsilon]$.)

For fixed $\rho \in [2, 2\vartheta]$, the function $\phi_\rho(t) := \frac{\rho}{\sqrt{t}}$ leads to

$$\varphi_\rho(x_\epsilon) = \int_0^1 \phi_\rho(b(t) - a(t)^T x_\epsilon)\ dt.$$

In the following, for simplicity, we use $\bar{t} = 0$ and ignore the constants. It then follows that

$$\varphi_\rho(x_\epsilon) = \int_0^1 \frac{\rho}{\sqrt{\epsilon + t^2}}\ dt = \rho \left[\log(t + \sqrt{\epsilon + t^2})\right]_0^1 \approx -\frac{\rho}{2}\log\epsilon \tag{12}$$

satisfies (11). Hence, for "small" $\epsilon > 0$ the barrier function $\phi_\rho(t)$ is suitable. For "large" values of ϵ, when $t - \bar{t}$ is small compared to ϵ for t in $[0,1]$, similar arguments as before lead to $\hat{\phi}_\rho(t) \approx -\frac{\rho}{2}\log t$ contrasting $\phi_\rho(t) = \frac{\rho}{\sqrt{t}}$ in (12). Here, the distinction of "large" and "small" depends on the particular problem (1), and without knowing the particular problem, it is not clear at which value of t the barrier function ϕ "should change" from the form ϕ_ρ to the form $\hat{\phi}_\rho$.

Moreover, if we consider a second case where we assume a degenerate behavior, we are led to another form of incompatibility: Assume that there is a point $\tilde{x}$ and a $\Delta\tilde{x}$ with $\tilde{x} + \lambda\Delta\tilde{x} \in \mathcal{S}$ iff $\lambda \geq -1$, and such that

$$b(t) - a(t)^T\tilde{x}_\epsilon = \tilde{f}(\epsilon) + \Theta(t - \tilde{t})^4$$

for $\tilde{x}_\epsilon = \tilde{x} + (\epsilon - 1)\Delta\tilde{x}$. Here, the differentiable function $\tilde{f}$ with $\tilde{f}(0) = 0$ and $\tilde{f}'(0) > 0$, as well as $\tilde{t} \in [0,1]$ are analogous as for x_ϵ. In analogy to our example following (10), this situation occurs when representing the constraint $x_2 \geq x_1^4$ in a semi-infinite form.

Neglecting additive constants, the bound (11) is true for $\varphi(\tilde{x}_\epsilon)$ as well. To illustrate the incompatibility, we assume $\bar{t} = \tilde{t} = 0$, $f(\epsilon) = \tilde{f}(\epsilon) = \epsilon$, and replace $\Theta(t)$ by t. (Constants do not change the arguments below.) Applying (11) to the points x_ϵ then leads to

$$\int_0^1 \phi(\epsilon + t^2)\ dt = -\Theta\log\epsilon \tag{13}$$

and when applying (11) to the points $\tilde{x}_\epsilon$ we obtain

$$\int_0^1 \phi(\epsilon + t^4)\ dt = -\Theta\log\epsilon.$$

For the second integral we substitute $u = t^2$ and $\frac{du}{dt} = 2t = 2\sqrt{u}$, and obtain

$$\int_0^1 \frac{1}{2\sqrt{u}}\phi(\epsilon + u^2)\ du = -\Theta\log\epsilon. \tag{14}$$

It is intuitive that (13) and (14) contradict each other, and using the monotonicity of ϕ, the argument can be made precise when comparing the integrals for small values of ϵ and $\tilde{\epsilon} = \epsilon^k$ with $k \in I\!N$. We omit the technical estimates.

The preceding discussion suggests that even if we prohibit the case that $b(t) + \epsilon$ is a linear combination of the $a_i(t)$, it is generally not possible to construct an integral self-concordant barrier function for (1) based on (3). Overall, we may conclude that it is unlikely that we will be able to identify a barrier function ϕ and a significantly large class of problems (1) for which (3) results in a self-concordant integral barrier function.

3. An implicit barrier function

The observation that the boundary of $\mathcal{S}$ is piecewise smooth when a and b are smooth leads to the conjecture that standard nonlinear optimization techniques can be modified to solve problem (1). Here, we compare the barrier function (3) with another approach for a barrier method based on a technique called "local reduction", see e.g. [5, 6]. We make the following assumption.

Assumption 2 *We assume throughout that $a(.)$ and $b(.)$ are sufficiently smooth functions.*

For a point $x \in \text{int}(\mathcal{S})$ let

$$\hat{T} = \hat{T}_x = \{\hat{t}_1, \ldots, \hat{t}_m\} \tag{15}$$

be the set of local minimizers of $b(t) - a(t)^T x$. We assume that $\hat{t}_i < \hat{t}_j$ for $i < j$. In (15), the value m may depend on x. Occasionally, we write $\hat{t}_i(x)$ and $m(x)$ instead of $\hat{t}_i$ and m to emphasize the dependence on x. With definition (15) we now state a second assumption used below.

Assumption 3 *We assume throughout that $m(x) \le \bar{m}$ is bounded independently of x.*

(This assumption implies for example, in the case of $\|a(t)\|_2 \equiv 1$ that the boundary of $\mathcal{S}$ does not touch any ball at more than finitely many points. This assumption is not essential for our approach; however, if it is violated, our presentation will become somewhat more complicated.)

Let again x be given. For $1 \le i \le m(x)$ we define

$$f_i(x) := a(\hat{t}_i(x))^T x - b(\hat{t}_i(x)).$$

(The definition implies $f_i(x) < 0$ for points $x \in \text{int}(\mathcal{S})$.) Let

$$d(t) := d_x(t) := b(t) - a(t)^T x,$$

so that $d(\hat{t}_i) = -f_i(x)$. In the following we omit the index i of $\hat{t} = \hat{t}_i$, and likewise we write shortly f instead of f_i. For $\hat{t} \in (0, 1)$ it follows

$$d'(\hat{t}) = b'(\hat{t}) - a(\hat{t})'^T x = 0$$

(since $\hat{t}$ is a local minimizer of d) and $d''(\hat{t}) \ge 0$.

For $d''(\hat{t}) > 0$ we consider small perturbations Δx and corresponding changes $\Delta\hat{t}$ in $\hat{t}$ with

$$\hat{t}(x + \Delta x) = \hat{t}(x) + \Delta\hat{t}.$$

From

$$\begin{aligned}
0 &= d'_{x+\Delta x}(\hat{t} + \Delta\hat{t}) \\
&= b'(\hat{t} + \Delta\hat{t}) - a'(\hat{t} + \Delta\hat{t})^T (x + \Delta x) \\
&\approx b'(\hat{t}) + b''(\hat{t})\Delta\hat{t} - a'(\hat{t})^T x - a'(\hat{t})^T \Delta x - a''(\hat{t})^T x \Delta\hat{t} \\
&= d'_x(\hat{t}) + d''_x(\hat{t})\Delta\hat{t} - a'(\hat{t})^T \Delta x \\
&= d''_x(\hat{t})\Delta\hat{t} - a'(\hat{t})\Delta x
\end{aligned}$$

follows that

$$\Delta\hat{t} = \frac{a'(\hat{t})\Delta x}{d''(\hat{t})} + o(\|\Delta x\|), \quad \text{or} \quad \nabla_x \hat{t}(x) = \frac{a'(\hat{t}(x))}{d''_x(\hat{t}(x))}.$$

From this we deduce

$$\nabla f(x) = a(\hat{t}(x)) + (a'(\hat{t}(x))^T x)\nabla_x \hat{t}(x) - b'(\hat{t}(x))\nabla_x \hat{t}(x) = a(\hat{t}(x))$$

and

$$\nabla^2 f(x) = a'(\hat{t}(x))\nabla_x \hat{t}(x)^T = \frac{a'(\hat{t}(x))\ a'(\hat{t}(x))^T}{d''_x(\hat{t}(x))}.$$

Note that the Hessian matrix is positive semidefinite implying that if f is twice differentiable then it is convex. Assume now that $\bar{x}$ is a point with $d''_{\bar{x}}(\hat{t}) = 0$ for some value $\hat{t}$.

For example, let $\bar{x}$ be a point for which there exists a Δx with $m(\bar{x}) - m(\bar{x} + \epsilon\Delta x) > 0$ for all sufficiently small $\epsilon > 0$. Then, there will be values $\hat{t}$ with $d''(\hat{t}) = 0$. In this case, f is not twice differentiable at $\bar{x}$ even when a and b are smooth. On the other hand, in this case, the boundary of $\mathcal{S}$ has a well defined curvature at the points $\hat{t}$ closest to $\bar{x}$; it is given by the inverse of the radius of the ball that approximates the boundary of $\mathcal{S}$ to second order. Thus, in the formula for $\nabla^2 f(x)$ we replace the term $d''(\hat{t}(x))$ by $\max\{\|a(\hat{t}(x))\|_2^2/d(\hat{t}(x)),\ d''(\hat{t}(x))\}$ to obtain a bounded approximation to $\nabla^2 f(x)$,

$$\nabla^2_{bounded}\ f(x) = \frac{a'(\hat{t}(x))\ a'(\hat{t}(x))^T}{\max\{\|a(\hat{t}(x))\|_2^2/d(\hat{t}(x)),\ d''(\hat{t}(x))\}}.$$

This allows us to define a new barrier function $\hat{\varphi}$ for $\mathcal{S}$,

$$\hat{\varphi}(x) := -\sum_{i=1}^{m(x)} \log(-f_i(x))$$

with derivative

$$\nabla\hat{\varphi}(x) = \sum_{i=1}^{m(x)} \frac{a(\hat{t}_i(x))}{-f_i(x)}$$

and approximate second derivative

$$\nabla^2_{bounded}\hat{\varphi}(x) = \sum_{i=1}^{m(x)} \left(\frac{a(\hat{t}_i(x))\ a(\hat{t}_i(x))^T}{f_i(x)^2} + \frac{a'(\hat{t}(x))\ a'(\hat{t}(x))^T}{\max\{\|a(\hat{t}(x))\|_2^2,\ -f_i(x)\cdot d''(\hat{t}(x))\}} \right).$$

We may now replace the barrier function φ of Section 2 either by $\hat{\varphi}$ or by a weighted sum of $\hat{\varphi}$ and φ.

We point out that there is a *qualitative difference* of the function φ and the function $\hat{\varphi}$ in that the vectors $a'(\hat{t})$ enter the second derivative of $\hat{\varphi}$ providing information about the curvature of the boundary of the feasible set $\mathcal{S}$.

While the above simple representation of the derivatives of f relies on the special form of the set $\Omega = \{t \mid t \in [0,1]\}$, more complicated sets Ω in higher dimensions allow the same technique (using the optimality conditions for higher dimensional vectors $\hat{t}$) but require more "patience" with evaluating the second derivative $\nabla^2_{bounded}\ f(x)$.

Obviously, with the above representation the infinitely many constraints of (1) have been converted to finitely many constraints. If the number m remains constant for the iterates generated by an interior-point algorithm we may anticipate that some of the nice practical properties of barrier function methods for finite optimization problems carry over to this new approach. If there are only few changes in $m(x)$ (corresponding to adding and dropping constraints in finite dimensional optimization) there may be a way to adapt an interior-point method accordingly.

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